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Analytical Prediction of the Influence of Polymer Additives on the Shear Drag of Bodies of Revolution

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Theme

THIS Synoptic discusses an analytical method for the determination of shear drag on a submerged body of revolution when a polymer additive is injected into the boundary layer. In the case of external flow, as for bodies of revolution, the polymer is normally injected into the boundary layer from storage facilities within the body. Since the polymer will diffuse through the boundary layer, the polymer concentration will vary both normal to the surface of the body and along the surface of the body. This analysis must account for the variation in drag reduction effectiveness of the polymer as a result of the change in polymer concentration.

In order to predict the drag reduction, it will first be necessary to predict polymer concentration. This calculation was performed by Fabula and Burns[1] for flow over a flat plate. The case of drag on a flat plate with a pressure gradient was presented in a paper by White,[2] but he assumed the polymer concentration to be constant. The analysis that will be presented in this report is a continuation of work initiated at the Naval Underwater Systems Center, Newport, R.I.

Contents

In order to calculate the shear drag on the body it is necessary to calculate the local wall shear stress and integrate this over the surface of the body. The local wall shear stress on the body is determined from the momentum integral equation for a body of revolution

$$\frac{d}{dx}\int_0^\delta u^2dy - U\frac{d}{dx}\int_0^\delta udy + \frac{1}{R}\frac{dR}{dx}\left[\int_0^\delta u^2dy - U\int_0^\delta udy\right] = \delta U\frac{dU}{dx} - \frac{g_c\tau_0}{\rho} \quad (1)$$

where ρ = density of fluid in boundary layer, u = velocity at some point within the boundary layer, and τ_0 = wall shear stress.

The geometry appropriate for Eq. (1) is indicated in Fig. 1. The abovementioned equation also depends on the assumption that δ/R is small.

In order to solve Eq. (1), the following steps must be taken:

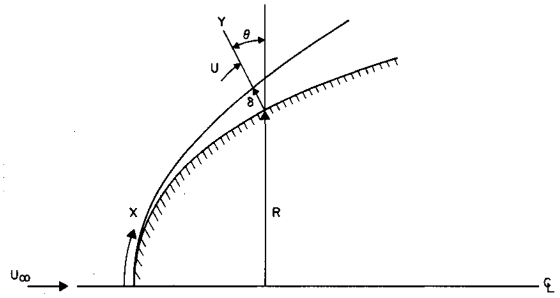

Fig. 1 Geometry for body of revolution.

1) Obtain R as a function of x from the known geometry for the body of revolution.

2) Determine U as a function of x from the potential flow solution for the body.

3) Assume u as a function of y or η ($\eta = y/\delta$) and the functional relationship should realistically account for axial pressure gradients and polymer addition.

Step 3 will normally also give a relation between $u^*[u^* = (g_c\tau_0/\rho)^{1/2}]$ and δ, and this is used to eliminate δ from Eq. (1). The result is an ordinary differential equation for u^* as a function of x. This equation is then integrated by some finite-differece technique.

The equation for the velocity profile is

$$\frac{u}{u^*} = 2.5\ \ln\left(\frac{\delta u^*}{\nu}\right) + 2.5\ \ln\eta + 5.5 - 0.6\frac{U\delta}{u^{*2}}\frac{dU}{dx} + \gamma\ \ln\left(\frac{u^*}{u_0^*}\right) \quad (2)$$

where ν = kinematic viscosity of fluid in boundary layer. An additional equation results from recognizing that, in Eq. (2), $u = U$ when $\eta = 1$. The value of γ in Eq. (2) depends on wall concentration and u_o^* is the u^* at which the polymer becomes effective. The wall concentration is obtained by integrating an assumed polymer concentration profile from the wall to an infinite distance normal to the wall. The wall concentration then is a function of the polymer injection rate.

If the wall concentration as a function of x is combined with Eqs. (1) and (2), the result can be numerically integrated and boundary-layer thickness, wall concentration, local shear, and total drag can be determined for a particular body. The mathematical model will be applied in a classical "half-body" which has an L/D of 10.

The body is assumed to be travelling submerged in water at a velocity of 30 fps. The polymer is injected from the body before the transition from laminar to turbulent flow. Two polymer injection rates are used: 1.85% of the base rate and 7.4% of the base rate.

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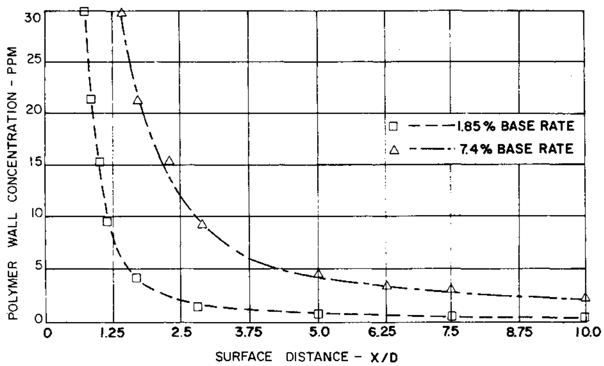

Fig. 2 Polymer wall concentration.

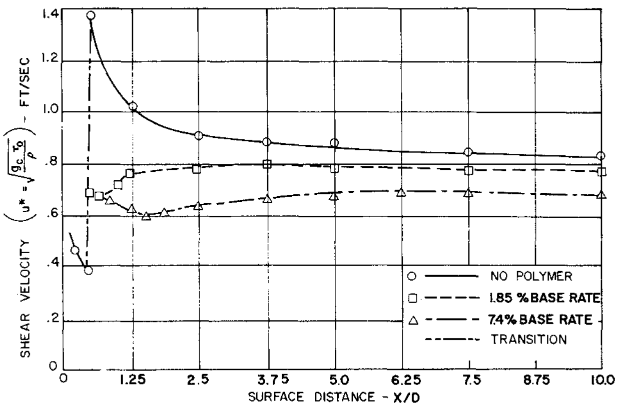

Fig. 3 Shear velocity.

The base rate is calculated by assuming the polymer wall concentration is 25 ppm at a location halfway along the body and that all the polymer is within the hydrodynamic boundary-layer thickness calculated without polymer. The transition from laminar to turbulent flow can be assumed to occur at the point of minimum pressure coefficient.

The results for the calculation of the polymer concentration at the wall are given by Fig. 2. The polymer concentration will also decrease, along the y-axis. The wall concentration is important because it determines the value of γ in the equation for the velocity profile. Since γ decreases if c_w is less than 25 ppm, it can be seen from Fig. 2 that even greater rates of polymer injection should be more effective in reducing shear drag.

The local wall shear stress as indicated by the shear velocity is shown in Fig. 3. The polymer is assumed not to be effective until the flow becomes turbulent, so there is no difference in the various curves until x/D is greater than 0.5. During the initial portion of turbulent flow, the results for polymer injection rate of 1.85% and 7.5% of the base rate are the same because the wall concentrations in both cases are above 25 ppm. The results with polymer and without polymer converge as the surface distance increases since the polymer wall concentration is continually decreasing.

The total shear drag can be obtained by graphically integrating the local shear stress over the surface of the body. There is a reduction in shear drag of 26% for a polymer injection rate of 1.85% of the base rate and 42% for 7.4% of base rate. The percent reduction in total drag would be less than this, as it would include the form drag which is assumed to be unaffected by the polymer.

The question might be asked as to whether the shear drag will continually be reduced as the amount of polymer is increased. The influence of the polymer depends on the wall concentration, but the polymer effect on drag does not change if the wall concentration is above 25 ppm. The maximum drag reduction will occur when sufficient polymer is injected to maintain the wall concentration above 25 ppm over the entire surface of the body. If additional amounts of polymer are injected there will be no further drag reduction.

Conclusions

1) Integral techniques can be used to determine the shear drag reduction on a body of revolution as a result of polymer injection into the boundary layer.

2) Increased rates of polymer injection increase the percent reduction in shear drag, providing the wall concentration does not stay above 25 ppm by weight along the entire body.

3) The finite-difference solution is stable for the "half-body" but can become unstable and "blow-up" on the latter portion of bodies of revolution that have a significantly varying pressure coefficient far downstream on the body.

References

[1]Fabula, A. G. and Burns, T. J., "Dilution in a Turbulent Boundary Layer with Polymeric Friction Reduction," presented at the AIAA 2nd Advanced Marine Vehicles and Propulsion Meeting, Seattle, Wash., 1969.

[2]White, F. M., "An Analysis of Flat Plate Drag with Polymer Additives," *Journal of Hydronautics*, Vol. 2, No. 4, Oct. 1968, pp. 181–186.